J. Fluid Mech. (2002), *vol.* 458, *pp.* 219–228.
DOI: 10.1017/S0022112002008017 Printed in the United Kingdom

Lewis-number effects on edge-flame propagation

By VEDHA NAYAGAM[1] AND F. A. WILLIAMS[2]

[1]National Center for Microgravity Research, NASA Glenn Research Center, Cleveland, OH 44135, USA

[2]Center for Energy Research, Department of Aerospace and Mechanical Engineering, University of California, San Diego, La Jolla, CA 92093-0411, USA

(Received 14 December 2001 and in revised form 24 January 2002)

Activation-energy asymptotics is employed to explore effects of the Lewis number, the ratio of thermal to fuel diffusivity, in a one-dimensional model of steady motion of edges of reaction sheets. The propagation velocity of the edge is obtained as a function of the relevant Damköhler number, the ratio of the diffusion time to the chemical time. The results show how Lewis numbers different from unity can increase or decrease propagation velocities. Increasing the Lewis number increases the propagation velocity at large Damköhler numbers and decreases it at small Damköhler numbers. Advancing-edge and retreating-edge solutions are shown to exist simultaneously, at the same Damköhler number, if the Lewis number is sufficiently large. This multiplicity of solutions has a bearing on potential edge-flame configurations in non-uniform flows.

1. Introduction

Highly strained diffusion flames experience extinction. Strain-rate variations in non-uniform flows can lead to extinctions at some locations while the diffusion flame remains intact at others. This may result in a diffusion flame with an edge. Mixing of fuel and oxidizer in the extinguished regions can lead to triple flames, which have been addressed in a number of publications (Kioni *et al.* 1993; Ruetch, Vervisch & Liñán 1995; Vervisch & Poinsot 1998). These are composed of a rich premixed flame and a lean premixed flame, with a diffuion flame trailing behind from the nose where they come together. The two premixed flames of the triple flame disappear, merging into a propagating edge of the diffusion flame, if the strain rate is high enough for the diffusive transport zones on each side of the diffusion flame to be comparable in size with the preheat zone of the stoichiometric premixed propagating nose. Detailed analyses of such edge flames in general necessitate considering the multidimensional, time-dependent conservation equations. Buckmaster (1996), however, identified a one-dimensional model that simplifies the analysis greatly and reveals many properties of edge flames. The present work explores further the implications of a model of this general type, addressing especially effects of the Lewis number of the fuel. In this respect, it differs from our earlier use of this model (Nayagam & Williams 2001*a*), which considered edge-curvature effects for Lewis number unity.

Experimental motivation for the present study stems from observations of rotating spiral edge flames in von Kármán swirling flows (Nayagam & Williams 2000). Under suitable conditions, slow injection of fuel through a spinning porous disk into air results in flat diffusion-flame spirals, separated from the disk by a distance only on the order of 1 mm, comparable with the corresponding premixed-flame preheat-zone thickness (Nayagam & Williams 2001*b*). Under other conditions, there are axisymmetric flame disks and holes, described and analysed previously in Nayagam & Williams (2001*a*). Propagation of the edge of a spiral into oncoming flow locally resembles edge-flame propagation. Explanations of spiral flames thus can use knowledge of edge-flame propagation velocities. As to be reported in a paper in preparation, the spirals are observed to be different for fuels of different molecular weights, thereby suggesting that the Lewis number of the fuel plays a role. For this reason, the present work addresses Lewis-number effects. Such effects have been studied previously in two-dimensional models (Daou & Liñán 1998; Thatcher & Dold 2000), but only to a limited extent in the one-dimensional model (Buckmaster 1996).

The spiral flames are observed near extinction conditions, in that a small decrease in the flow rate of the fuel completely extinguishes the flame. A mixture fraction, defined to be unity in the fuel feed stream and zero in the air, already has a small value, of order 0.1, at the external surface of the disk. The dominant heat loss from the diffusion flame therefore occurs by conduction through the steep temperature gradient from the flame to the surface of the disk. This tends to cause oxygen leakage through the diffusion flame, even on the basis of one-step activation-energy asymptotics, independent of detailed chemistry. Liñán's (1974) premixed-flame regime of the diffusion flame therefore appears to be more applicable than the diffusion-flame regime addressed by Buckmaster (1996). The analysis is presented here for the premixed-flame regime, although results are quoted also for the diffusion-flame regime. Like Buckmaster (1996), we take the Lewis number of the oxidizer to be unity, primarily because we believe that to be appropriate for the spiral-flame experiments in air. Unlike Buckmaster (1996), however, we do not presume the fuel Lewis number to be near unity in the detailed development, since we find that relaxing this restriction adds a new dimension to the analysis and interpretations.

The one-dimensional edge-flame model resembles a premixed-flame model with distributed heat loss. Such heat loss leads to flame extinction (Joulin & Clavin 1979; Williams 1985). The primary difference is that, in addition to losing heat algebraically from the side, the edge flame gains fuel algebraically from the side. This is a qualitatively strong difference that produces excess enthalpy when the fuel Lewis number differs from unity. There is thus similarity to flames in excess-enthalpy burners in which enhanced conduction of heat increases flame temperatures (Takeno, Sato & Hase 1981). The following analysis may help to expose these various similarities and differences.

2. Formulation

Steady propagation of the edge of the flame in the x-direction is considered in figure 1. The edge is located at $x = 0$, and the diffusion flame extends to $x = \infty$. In terms of density, ρ, the component, v, of gas velocity in the x-direction and the component, u, of gas velocity in the direction, z, normal to the diffusion-flame sheet, integration of the equation of mass conservation across the reaction sheet results in

$$\frac{\mathrm{d}(\rho v)}{\mathrm{d}x} = \frac{-[(\rho u)_+ - (\rho u)_-]}{t}, \tag{2.1}$$

where t denotes the thickness of the reaction sheet, which is of order εa, a being the thickness of the diffusion layer between the sheet and the boundary and ε the small parameter, the reciprocal of the non-dimensional activation energy. Here ρv represents an average over the thin reaction zone, as will the other dependent variables

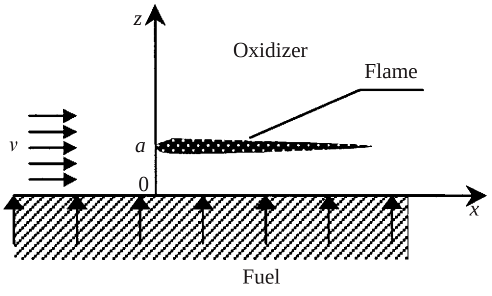

FIGURE 1. Schematic illustration of the problem.

introduced below. The subscripts + and − identify conditions at the values z_+ and z_- of z, just above and just below the reaction sheet, respectively. The negative right-hand side of (2.1) describes the effects of streamline divergence as the fluid approaches the hot flame. This is neglected, ρv being treated as constant. With this approximation, the correspondingly integrated fuel and energy conservation equations are

$$\rho v \frac{\mathrm{d}Y}{\mathrm{d}x} - \frac{\mathrm{d}}{\mathrm{d}x}\left(\rho D \frac{\mathrm{d}Y}{\mathrm{d}x}\right) = -w + \frac{\rho D}{at}(1 - Y), \tag{2.2}$$

$$\rho v \frac{\mathrm{d}T}{\mathrm{d}x} - \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\lambda}{c_p} \frac{\mathrm{d}T}{\mathrm{d}x}\right) = \frac{q}{c_p} w - \frac{\lambda / c_p}{at}(T - T_0). \tag{2.3}$$

Here T denotes temperatures and Y the ratio of the fuel mass fraction to its value in the fuel stream. The mass rate of consumption of fuel is given by the Arrhenius form

$$w = \rho A Y \mathrm{e}^{-E/RT} \tag{2.4}$$

with E representing the activation energy, R the universal gas constant and A a characteristic reciprocal-time rate prefactor. For a bimolecular reaction, A is proportional to both pressure and the oxidizer mass fraction, the conservation equation for which need not be addressed in the premixed-flame regime for the one-dimensional model.

In (2.2) and (2.3), the diffusion coefficient of the fuel is D, and the coefficient of thermal conductivity is λ. The specific heat at constant pressure, c_p, is assumed constant, and q denotes the heat released per unit mass of fuel consumed. The last term in (2.2) and (2.3) is obtained by evaluating the gradient at $z = z_-$ after integrating across the sheet; gradients at $z = z_+$ are neglected in comparison. The wall temperature is T_0, profiles are assumed linear in the diffusion layer, and the products ρD and λ / c_p are taken constant. The model of Buckmaster (1996) is less explicit, the product at being a^2, the square of a representative side diffusion length, which becomes $a\sqrt{\varepsilon}$ in the present formulation.

The boundary conditions for (2.2) and (2.3) are obtained by setting $w = 0$ for $x \to -\infty$, giving $Y \to 1$ and $T \to T_0$ as $x \to -\infty$, and vanishing of the right-hand sides in the diffusion flame as $x \to +\infty$. This last condition requires the non-dimensional temperature $\theta = c_p T/q$ to assume the value $\theta_\infty = \theta_0 + (1 - Y_\infty)/L$ as $x \to \infty$, where the Lewis number, of order unity, is $L = \lambda/(\rho D c_p)$, and the subscript ∞ identifies conditions at $x = \infty$.

A non-dimensional excess enthalpy, Z, can be defined by the equation

$$\theta = \theta_0 + (1 - Y)/L + Z. \tag{2.5}$$

A non-dimensional independent variable, s, and propagation velocity, V, are defined as $s = x/\sqrt{at}$, and $V = v(\rho c_p/\lambda)\sqrt{at}$, and the relevant Damköhler number is

$$\Delta = (\rho c_p/\lambda) at A \mathrm{e}^{-E/RT_f}, \tag{2.6}$$

where T_f is a characteristic premixed-flame temperature, to be determined. The small parameter of expansion is taken to be

$$\varepsilon = LRT_f^2/[E(q/c_p)], \tag{2.7}$$

the reciprocal of the Zel'dovich number, and solutions are sought for which V is of order unity, with the ordering of Δ not yet specified. Equations (2.2) and (2.3) then reduce to

$$LV\frac{\mathrm{d}Y}{\mathrm{d}s} - \frac{\mathrm{d}^2Y}{\mathrm{d}s^2} - (1-Y) = -L\Delta Y \exp\left(\frac{L}{\varepsilon}\left(\frac{\theta-\theta_f}{\theta/\theta_f}\right)\right), \tag{2.8}$$

$$\frac{\mathrm{d}^2Z}{\mathrm{d}s^2} - V\frac{\mathrm{d}Z}{\mathrm{d}s} - Z = V\left(\frac{L-1}{L}\right)\frac{\mathrm{d}Y}{\mathrm{d}s}. \tag{2.9}$$

Boundary conditions for (2.8) and (2.9) are $Y \to 1$ and $Z \to 0$ as $s \to -\infty$ and $Y \to Y_\infty$ and $Z \to 0$ as $s \to +\infty$, according to the previous conditions. In (2.8), $\theta_f = c_pT_f/q$, and $\theta_f = \theta_0 + (1-Y_f)/L + Z_f$ according to (2.5), where the subscript f identifies outer-zone variables evaluated at the premixed-flame reaction zone.

3. Solution

The premixed reaction zone is placed at $s = 0$. The value of T_f is to obey the restriction that, in the region $s < 0$, $\theta < \theta_f$, so that the reaction term in (2.8) is exponentially small. In this region, (2.8) and (2.9) are both linear and have as their general solution, subject to the boundary conditions at $s = -\infty$,

$$Y = 1 - (1-Y_f)\mathrm{e}^{\beta s}, \tag{3.1}$$

$$Z = Z_f\mathrm{e}^{\alpha s} + (\mathrm{e}^{\alpha s} - \mathrm{e}^{\beta s})(1-Y_f)/L, \tag{3.2}$$

where

$$\alpha = (V + \sqrt{V^2+4})/2, \ \beta = (LV + \sqrt{L^2V^2+4})/2. \tag{3.3}$$

Possible partial-burning solutions, for which Y_f is of order unity, are not addressed. The value of Y_f is instead assumed no larger than order ε, that is, $Y_f = \varepsilon y_f$, where y_f may be of order unity. In the reaction zone at $s = 0$ it is then appropriate to introduce stretchings

$$y = Y/\varepsilon, \ \eta = \beta s/\varepsilon + \eta_f, \tag{3.4}$$

where the order-unity factor β is for later convenience in matching, and where η_f is a constant of order unity. Presuming Z_f to be of order unity, we deduce from (2.9) that, to leading order, both Z and $\mathrm{d}Z/\mathrm{d}s$ are continuous across the reaction zone, while there is a jump in $\mathrm{d}^2Z/\mathrm{d}x^2$ proportional to the jump in $\mathrm{d}Y/\mathrm{d}s$. Through terms of order ε, then, within the reaction zone, $Z = Z_f + \varepsilon Z_f'(\eta - \eta_f)/\beta$, where

$$Z_f' = \alpha Z_f + (\alpha - \beta)/L \tag{3.5}$$

from the derivative of (3.2). Substitution of (2.5), (3.4) and (3.5) into (2.8) then yields, to leading order, the problem

$$2\frac{\mathrm{d}^2y}{\mathrm{d}\eta^2} = y\mathrm{e}^{-(y+m\eta)}, \quad \frac{\mathrm{d}y}{\mathrm{d}\eta} \to -1 \quad \text{as} \quad \eta \to -\infty, \quad \frac{\mathrm{d}y}{\mathrm{d}\eta} \to 0 \quad \text{as} \quad \eta \to +\infty, \tag{3.6}$$

provided that Δ is of order ε^{-2}, η_f is chosen such that

$$\Delta = \beta^2(2\varepsilon^2 L)^{-1}\mathrm{e}^{-(y_f+m\eta_f)}, \tag{3.7}$$

and m is defined as

$$m = -LZ_f'/\beta. \tag{3.8}$$

The derivative of (3.1) was used in deriving the matching condition for $\eta \to -\infty$. Equation (3.6) is the problem describing the inner structure in Liñán's premixed-flame regime, and the definitions have been selected to obtain exactly the same form, so that available solutions (Liñán 1974) can be used most readily.

These results enable the complete solution for the excess enthalpy to be obtained at leading order. In the region $s > 0$, the left-hand side of equation (2.9) is of higher order, and the solution having $Z \to 0$ as $s \to \infty$ is then

$$Z = Z_f\mathrm{e}^{-\gamma s}, \tag{3.9}$$

where

$$\gamma = (\sqrt{V^2+4} - V)/2, \tag{3.10}$$

and the continuity of Z at $s = 0$ has been employed. The continuity of $\mathrm{d}Z/\mathrm{d}s$ implies from the derivative of (3.9) that $Z_f' = -\gamma Z_f$, which may be used in equation (3.5) to show with the aid of (3.3) and (3.10) that

$$Z_f = (\beta - \alpha)/(L\sqrt{V^2+4}). \tag{3.11}$$

By making use of equations (3.1), (3.2), (3.3), (3.9) and (3.10), it can be shown that the jump condition for $\mathrm{d}^2Z/\mathrm{d}s^2$, implied by (2.9), is satisfied identically as a consequence of (3.11), and equation (3.8) becomes

$$m = \gamma(\beta - \alpha)/(\beta\sqrt{V^2+4}). \tag{3.12}$$

These results enable the solution for the normalized fuel mass fraction Y to be obtained downstream ($s > 0$) to leading order. Equation (2.5) indicates that, in this region, to leading order $\theta = \theta_\infty + Z$, with Z given by (3.9). From (3.11), then,

$$\theta_f = \theta_\infty + \frac{\beta m}{\gamma} = \theta_\infty + \frac{\beta - \alpha}{L\sqrt{V^2+4}}. \tag{3.13}$$

Substitution of these results into (2.8), with the realization that Z (and therefore θ) is evolving on a length scale such that changes in s are of order unity, demonstrates that the largest terms in the expansion for small ε, with Y of order ε or smaller, are provided by the right-hand side and by unity on the left-hand side. Hence the dominant balance is that of reaction and side diffusion in the diffusion flame, with a variable flame temperature imposed by the convective–diffusive balance of the excess enthalpy. These substitutions into (2.8) produce, to leading order,

$$Y = \left(\frac{1}{L\Delta}\right)\exp\left[\frac{-(L/\varepsilon)\theta_f(1-\mathrm{e}^{-\gamma s})}{(\theta_f/\theta_\infty - 1)^{-1} + \mathrm{e}^{-\gamma s}}\right], \tag{3.14}$$

which is exponentially small in ε for $s > 0$ whenever $\theta_f - \theta_\infty$ is larger than order ε. At $s = 0$, Y is of order ε^2 in this region according to (3.7) and (3.14). Similarly, as $s \to \infty$, in terms of a redefined Damköhler number and small parameter ε, based on T_∞ instead of T_f, Y is of order ε^2 when the Dahmköhler number is of order ε^{-2}.

The propagation velocity V is obtained from the solution for the premixed-flame regime (Liñán 1974), which relates Δ to m. The solution provides n as a function of m, where n denotes the limiting value of $y+\eta$ as $\eta \to -\infty$. The matching condition for $\eta \to -\infty$ is then found from (3.1) and (3.4) to require

$$y_f + \eta_f = n. \tag{3.15}$$

To determine the solution at this order completely, it is necessary to consider the solutions in the outer zones at order ε, for example by replacing Z_f by $Z_f + \varepsilon z_f$ and imposing continuity of the value and slope of Z across the reaction zone at this order. It is thereby found that the jump condition for $\mathrm{d}^2Z/\mathrm{d}s^2$ and matching can be satisfied at this order in different ways. One possibility is to put $z_f = 0$, implying that $y_f = 0$, that is Y_f is smaller than order ε, namely of order ε^2, to match to the downstream solution. From this result and (3.15), equation (3.7) then becomes

$$\Delta = \beta^2(2\varepsilon^2 L\mathrm{e}^{mn})^{-1}. \tag{3.16}$$

With the problem posed as specifying V at order ε and finding Δ to leading order, (3.16) results. In view of (3.3) and (3.10), equation (3.12) relates m to V and L, and from the known (Liñán 1974) dependence of mn on m, equation (3.16) then relates Δ to V and L. The relationships are somewhat involved since, from (2.7), $\varepsilon = \theta_f^2 LRq/(c_pE)$, with θ_f dependent on V and L according to (3.13), and, similarly, from (2.6),

$$\Delta = \Delta_\infty \exp\left[\frac{E}{RT_\infty}\left(\frac{\theta_f - \theta_\infty}{\theta_f}\right)\right], \tag{3.17}$$

where Δ_∞ is defined by replacing T_f by T_∞ in (2.6). It is therefore of interest to investigate limiting cases.

4. Results

From (2.5), (3.1), (3.2) and (3.11), the preheat-zone solution for temperature is

$$\theta = \theta_0 + \frac{1}{L}\left(1 + \frac{\beta - \alpha}{\sqrt{V^2+4}}\right)\mathrm{e}^{\alpha s}, \tag{4.1}$$

which is monotonic, with $\theta_f > \theta_0$ for all values of V and L. Downstream, however,

$$\theta = \theta_\infty + \left(\frac{\beta - \alpha}{L\sqrt{V^2+4}}\right)\mathrm{e}^{-\gamma s}, \tag{4.2}$$

which has $\theta_f > \theta_\infty$ only for $\beta > \alpha$, that is, for $L > 1$ if $V > 0$ and for $L < 1$ if $V < 0$. Positive excess enthalpy is generated for Lewis numbers greater than unity when $V > 0$ and for Lewis numbers less than unity when $V < 0$. If either $L = 1$ or $V = 0$, then there is no excess enthalpy, and $\theta_f = \theta_\infty$. According to (3.11) and (3.12), the excess enthalpy is positive for $m > 0$ and negative for $m < 0$, as it must be (Liñán 1974). Stable propagation is anticipated for $m < 0$, heat from the diffusion flame driving the premixed edge flame, but instability is anticipated as m becomes sufficiently positive (Peters 1978). This expectation is only partially qualitatively consistent with existing (Buckmaster 1996) stability results derived for Lewis numbers near unity; stability is not addressed in detail in the present work.

Although various expansions can be developed for large and small values of L, it is of greatest interest to consider Lewis numbers near unity, putting $L = 1 + \varepsilon\ell$ with

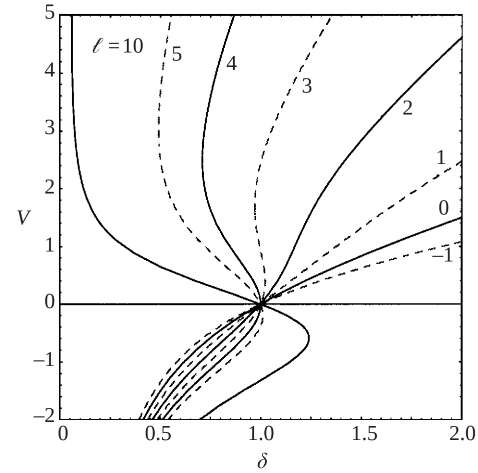

FIGURE 2. Edge velocity as function of Damköhler number for various ℓ values.

ℓ of order unity. In this limit,

$$\theta_f - \theta_\infty = Z_f = \frac{\varepsilon \ell V}{2\sqrt{V^2+4}}\left(1 + \frac{V}{\sqrt{V^2+4}}\right), \tag{4.3}$$

$$m = \frac{\varepsilon \ell V}{2\sqrt{V^2+4}}\left(1 - \frac{V}{\sqrt{V^2+4}}\right), \tag{4.4}$$

both of which are small. Expansions of (3.16) and (3.17) then provide, to leading order,

$$V + \sqrt{V^2+4} = 2\varepsilon_\infty \sqrt{2\Delta_\infty}\exp\left[\left(\frac{\ell V}{4\sqrt{V^2+4}}\right)\left(1 + \frac{V}{\sqrt{V^2+4}}\right)\right], \tag{4.5}$$

with the definition $\varepsilon_\infty = T_\infty^2 R/[E(q/c_p)]$. Equation (4.5) is finally a single expression for V as a function of ℓ and of

$$\delta = \varepsilon_\infty \sqrt{2\Delta_\infty} = \frac{T_\infty^2 R}{E(q/c_p)}\sqrt{\frac{2\rho c_p a t A}{\lambda}}\mathrm{e}^{-E/(2RT_\infty)}. \tag{4.6}$$

Figure 2 shows the dependence of V on δ for various values of ℓ, according to (4.5).

For $\ell = 0$, Lewis number of unity, (4.5) reduces to

$$V = \delta - \delta^{-1}, \tag{4.7}$$

which goes to infinity as $\delta \to \infty$ and minus infinity as $\delta \to 0$, and vanishes at $\delta = 1$. For $\ell \neq 0$ (4.5) is more complicated. When V is large and positive, (4.5) becomes

$$V = \delta \mathrm{e}^{\ell/2}, \tag{4.8}$$

showing that (figure 2), Lewis numbers greater than unity increase V, while Lewis numbers less than unity decrease it, in accord with the effect of Lewis number on excess enthalpy. When V is negative and large in magnitude, an expansion of (4.5) gives

$$V = -(1/\delta)\mathrm{e}^{\ell/(2V^2)}, \tag{4.9}$$

which again shows that Lewis numbers greater than unity increase the magnitude of

V, while Lewis numbers less than unity decrease it, also seen in figure 2. This again is consistent with influences of the excess enthalpy, since in this case Lewis numbers less than unity correspond to positive excess enthalpy, which may be expected to enhance propagation of the flame edge, that is, retard the rate at which the edge retreats. Unlike the situation for positive V, however, the effect of the Lewis number is seen from (4.9) to become negligible as the magnitude of V increases, yielding the limiting value $V = -1/\delta$, corresponding to the last term in (4.7) for $\delta \ll 1$, independent of the value of ℓ. Rapidly retreating edges prevent buildup of excess enthalpy.

Behaviours at smaller values of V, that is, for δ closer to unity, are more interesting. Expanding the functions of V in (4.5) for small values of V produces the equation

$$V + 2 = 2\delta e^{V\ell/8} \approx 2\delta(1 + V\ell/8). \tag{4.10}$$

The result that $V = 0$ when $\delta = 1$, as in (4.7), is seen to apply for all Lewis numbers, but for δ near unity, (4.10) becomes

$$V(1 - \ell/4) = 2(\delta - 1), \tag{4.11}$$

which reverses its behaviour for $\ell > 4$. When $\ell < 4$, the increase in V with increasing δ, at a rate that increases with increasing ℓ, persists. Increasing Lewis numbers, for Lewis numbers near unity, increase the magnitude of the propagation velocity, irrespective of whether it is positive or negative, that is, irrespective of whether δ exceeds or is less than unity. For $\ell > 4$, however, V decreases with increasing δ for δ near unity, at a rate that decreases with increasing ℓ. The propagation velocity becomes a non-monotonic function of the Damköhler number, there being three values of V for each value of δ in a range between a minimum $\delta < 1$ and a maximum $\delta > 1$. When $\delta < 1$, two values are positive and one negative, while for $\delta > 1$ two are negative and one positive (figure 2). Static stability reasoning suggests that the middle solution is unstable; for example, for $\delta < 1$, a small increase of V above the smaller of the two positive solutions increases the excess enthalpy, thereby tending to increase V further. It therefore appears that, when $\ell > 4$, if $\delta_{min}(\ell) < \delta < \delta_{max}(\ell)$ (with $\delta_{min}(\ell) < 1$, $\delta_{max}(\ell) > 1$, $\delta_{max}(\ell) \to 1$ as $\ell \to 4$, $\delta_{min}(\ell) \to 0$ and $\delta_{max}(\ell) \to \infty$ as $\ell \to \infty$), there are two statically stable edge-flame solutions for each ℓ, one for an advancing edge and the other for a retreating edge. Figure 2 shows, further, that this three-solution type of behaviour persists for $\ell < 4$, until $\ell = 2.4$, but over a decreasing range of δ, and now with all three solutions for V positive.

5. Discussion

The preceding analysis has addressed edge flames on the basis of a one-dimensional model. The resulting edges advance at large Damköhler numbers and retreat at small Damköhler numbers, as demonstrated previously (Buckmaster 1996). The Lewis number affects the rates of advance or retreat mainly by its influence on the excess enthalpy at the edge. For sufficiently large Lewis numbers, the dependence of the propagation velocity of the edge on the Damköhler number is not monotonic; there is a region about a propagation velocity of zero (an edge that is stationary with respect to the gas) over which the steady propagation velocity decreases with increasing Damköhler number, and the steady solution is likely to be unstable. Under these conditions there appear to be two stable steady solutions, one for an advancing edge and the other for a retreating edge. These solutions may apply to the leading and trailing edges observed (Nayagam & Williams 2000) for spiral flames in von Kármán swirling flows.

Although the full analysis was completed only for Lewis numbers near unity, the general formulas in the solution apply for arbitrary Lewis numbers. The inner structure of the steadily propagating edge in general was shown to be that of Liñán's premixed-flame regime (Liñán 1974), with heat loss away from the edge into the fresh mixture. There may be either heat loss or heat gain from the diffusion flame. As the Lewis number approaches unity, the loss or gain from the diffusion flame becomes small, producing the inner structure of an ordinary propagating adiabatic flame. The fuel concentration in the reaction zone at the edge exceeds its value in the diffusion flame, thereby tending to increase the chemical reaction rate there. When $\theta_f > \theta_\infty$, the Arrhenius factor also is larger at the edge, so that it is the most active part of the flame under these conditions. Since the full implications of the analysis were explored only for Lewis numbers near unity, it would be of interest to investigate further the edge-flame properties for general Lewis numbers on the basis of the solutions that have been obtained here.

Although the analysis was performed only for the premixed-flame regime of the downstream diffusion flame, similar results would be obtained for the diffusion-flame regime. The only difference would be that the oxidizer concentration also would vary in the inner reaction zone at the edge. A premixed-flame regime would still apply there, but in the reaction term in (3.6), for example, y would become y^2 in front of the exponential. Specific orderings would be affected. For example, for Lewis numbers near unity, fuel and oxidizer concentrations in the diffusion flame away from the edge are of order $\varepsilon\sqrt{\varepsilon}$ for the diffusion-flame regime (Buckmaster 1996), while the fuel concentration there is of order ε^2 for the premixed-flame regime analysed here. Since only these details are affected, the general characteristics of the solution do not depend on the specific assumptions of the problem. The present results would therefore apply to edges of both diffusion flames and premixed flames. Consideration of the diffusion-flame regime with an oxidizer Lewis number different from unity could, however, reveal different Lewis-number effects.

6. Conclusions

This study has shown how to describe steady edge-flame propagation for general Lewis numbers. It has demonstrated important influences of excess enthalpy. For a Lewis number of unity, there is no excess enthalpy, and the edge advances at high Damköhler numbers and retreats at low Damköhler numbers. The Damköhler number corresponding to a stationary edge is independent of the Lewis number. For all Lewis numbers less than unity, advancing edges develop negative excess enthalpy, exhibiting temperatures less than those of the downsteam planar flame. Consequently they propagate slower than they would if the Lewis number were unity. On the other hand, retreating edges develop positive excess enthalpy, exhibiting temperatures above those of the upstream flame. This causes a slower retreat than would occur if the Lewis number were unity. For Lewis numbers slightly greater than unity, advancing edges experience positive excess enthalpy and advance more rapidly, while retreating edges have negative excess enthalpy and retreat more rapidly, than they would if the Lewis number were unity. The competition between fuel diffusion and conductive heat loss is responsible for these effects. Advancing edges experience fuel depletion by transverse diffusion ahead of the edge, at relative rates that increase with decreasing Lewis numbers, while receding edges have conductive heat-loss rates that increase proportionally with increasing Lewis numbers.

There are conditions for sufficiently large Lewis numbers under which both advancing and receding edges may exist with the same chemical reaction-rate parameters. This non-monoticity in the dependence of the Damköhler number on the propagation velocity is a consequence of the interplay between heat conduction and fuel diffusion for weakly diffusing fuels in strongly conducting gases. Three solutions exist for the steady propagation velocity at a given Damköhler number in this intermediate range, the middle one of which is quite likely to be unstable. It would be worthwhile to perform stability analyses, allowing not only for pulsating instability but also for non-planar instability involving waves propagating along the flame edge and wrinkling it. From results for instabilities of planar premixed flames (Joulin & Clavin 1979), wrinkled cellular-flame instabilities are expected for Lewis numbers sufficiently less than unity. The same results suggest that travelling waves or pulsations may occur at larger Lewis numbers, especially those for which the multiplicity of propagation velocities exhibits both rapidly advancing and retreating edges.

The existence of stable steady advancing and retreating edges at the same Damköhler number may have a bearing on behaviours of experimentally observed edge flames. Spiral flames in von Kármán swirling flows possess steadily propagating well-defined leading and trailing edges (Nayagam & Williams 2000). Equating the propagation velocity to the flow velocity at each of these edges may enable the shape of the spiral to be calculated. The positive and negative propagation velocity may apply to the leading and trailing edge, respectively. Further research therefore seems warranted, investigating relationships of predictions of this simplified model to experimental measurements of edge flames.

This work was supported by the NASA Microgravity Combustion Science Program.

REFERENCES

Buckmaster, J. 1996 Edge-flames and their stability. *Combust. Sci. Tech.* **115**, 41–68.

Daou, J. & Liñán, A. 1998 The role of unequal diffusivities in ignition and extinction fronts in strained mixing layers. *Combust. Theory Modelling* **2**, 449–477.

Joulin, G. & Clavin, P. 1979 Linear stability analysis of nonadiabatic flames: diffusional-thermal model. *Combust. Flame* **35** 139–153.

Kioni, P. N., Rogg, B., Bray, K. N. C. & Liñán, A. 1993 Flame spread in laminar mixing layers: the triple flame. *Combust. Flame* **95**, 276–279.

Liñán, A. 1974 The asymptotic structure of counterflow diffusion flames for large activation energies. *Acta Astronautica* **1**, 1007–1039.

Nayagam, V. & Williams, F. A. 2000 Rotating spiral edge flames in von Kármán swirling flows. *Phys. Rev. Lett.* **84**, 479–482.

Nayagam, V. & Williams, F. A. 2001*a* Diffusion-flame dynamics in von Kármán boundary layers. *Combust. Flame* **125**, 974–981.

Nayagam, V. & Williams, F. A. 2001*b* Pattern formation in von Kármán Swirling Flows. *Sixth Intl Microgravity Combustion Workshop*, NASA/CP-2001-210826, pp. 97–100.

Peters, N. 1978 On the stability of Liñán's premixed flame regime. *Combust. Flame* **33**, 315–318.

Ruetsch, G. R., Vervish, L. & Liñán, A. 1995 Effects of heat release on triple flames. *Phys. Fluids* **7**, 1447–1454.

Takeno, T., Sato, K. & Hase, K. 1981 A theoretical study on excess enthalpy flame. *Proc. Combust. Inst.* **18**, 464–472.

Thatcher, R. W. & Dold, J. W. 2000 Edges of flames that do not exist: flame-edge dynamics in a non-premixed counterflow. *Combust. Theory Modelling* **4**, 435–457.

Vervisch, L. & Poinsot, T. 1998 Direct numerical simulation of non-premixed turbulent flames. *Annu. Rev. Fluid Mech.* **30**, 655–691.

Williams, F. A. 1985 *Combustion Theory*, 2nd Edn, pp. 271–284. Addison-Wesley.